A SYNOPSIS

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"On Some Instability Problems in Ferromagnetic Configurations in Porous and Non-Porous Medium and Thermohaline Configurations in Porous Medium"

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1. INTRODUCTION

Fluids exhibit deformation under the action of forces. If the deformation in the material increases continually without limit under the action of shearing forces, however small, the material is called a **fluid**. This continuous deformation under the action of forces compels the fluids to flow and this tendency of fluids is called **fluidity**.

Fluids are classified as liquids and gases. Liquids have strong intermolecular forces whereas the gases experience weak intermolecular forces. As a result of these, the liquids are incompressible fluids and the gases are highly compressible fluids. For the velocities which are not comparable with the velocity of sound, the effect of compressibility on atmospheric air can be neglected and it may be considered to be a liquid and in this sense it is called incompressible air.

Fluid dynamics is the science in which we study the properties of fluids in motion. Its study is of great practical importance. Some practical situations where fluid dynamics plays an important role are lubrication, flight of aeroplanes, ship science, meteorology, the influence of wind upon building structures, ground water seepage and the use of pipelines, pumps and turbines. The flow of fluid affects each one of us throughout our lives.

1.1. HYDRODYNAMIC STABILITY

In the stability analysis of a fluid flow problem, infinitesimal perturbations are applied on basic state (equilibrium) of a flow system and it is determined whether the equations of motion demand that the perturbations should grow or decay w. r. t. time. Stability analysis is of two types namely linear stability analysis and non-linear stability analysis. In the linear stability analysis we examine the stability of the flow w.r.t. the infinitesimally small perturbations and thus ignore from the governing perturbation equations all those terms which are of second or higher order in the perturbation quantity and / or their derivatives. On account of the drastic amplifications resorted to in the linear instability analysis, by dropping quadratic and higher order terms in perturbation and / or their derivatives, there are obvious limitations of their results obtained through such an analysis. Yet, the linear instability analysis has been found to be useful in providing results in many stability problems which are almost in exact agreement with the experimental predictions. In the linear instability analysis of the problem we examine only the initial behavior of the disturbances. In the non-linear instability analysis one has to solve a

system of non-linear partial differential equations. On account of the difficulties in solving non-linear differential equations we find that only a few studies are available which are devoted to non-linear system as compared to linear systems.

In the linear stability analysis characterization of the marginal state and the determination of amplitude of perturbation just past the marginal state are two of the important problems apart from the solution of the governing equations.

States of marginal stability can be one of two kinds:

- in which the amplitude of a small disturbance can grow or be damped aperiodically, or
- in which it can grow (or be damped) by oscillations of increasing (or decreasing) amplitude.

In the former case, the transition from stability to instability takes place via a marginal state exhibiting **stationary patterns** of motions. In the latter case, the transition takes place via a marginal state exhibiting oscillatory motions with a certain definite characteristic frequency.

If at the onset of instability, a stationary pattern of motion prevails, then one says that **principle of exchange of stabilities** is valid and that instability sets in as a stationary cellular convection or secondary flow. On the other hand, if at the onset of instability oscillatory motion prevails, then one says that one has a case of **overstability**.

1.2. THERMAL CONVECTION (OR BÉNARD PROBLEM)

Thermal instability often arises when a fluid is heated from below. When the temperature difference across the layer is large enough, the stabilizing effects of viscosity and thermal conductivity are overcome by the destabilizing buoyancy, and an overturning instability ensues as thermal convection. The problem of onset of convection in such a layer of fluid confined between two infinite horizontal planes is also known as Benard problem and has been extensively investigated both experimentally and theoretically. The Benard problem was studied mathematically for the first time by Lord Rayleigh (1916) for the idealized case of two free boundaries with a linear temperature gradient. He assumed that the amplitude of the motion was infinitesimal, so that the equation could be linearized. Rayleigh's theory shows that the gravity dominated thermal instability in a layer heated from below depends upon the Rayleigh number which is proportional to the uniform temperature difference of the layers of the concerning

liquid. When a horizontal layer of fluid of uniform density is subjected to an adverse temperature gradient by heating it from below, the fluid at the bottom become lighter than the fluid at the top and thus it becomes a *top-heavy arrangement,* which is potentially unstable and instability can set in only when the adverse temperature gradient exceeds certain critical value. Rayleigh showed that what decides the stability, or otherwise of a layer of fluid heated from below is the non-dimensional parameter called as the Rayleigh number and is given by $R = \frac{g\alpha\beta\, d^4}{\kappa_0 \upsilon_0}$, where g denotes the acceleration due to gravity, $\beta \left(= \left|\frac{dT}{dz}\right|\right)$, the uniform adverse temperature gradient which is maintained, d the depth of layer, α, κ_0 and υ_0 are coefficients of volume expansion, thermometric conductivity and kinematic viscosity respectively. Rayleigh further showed that instability must set in when R exceeds a certain critical value R_c and stationary patterns of motions must prevail when R just exceeds R_c .

1.3. THERMOHALINE INSTABILITY (OR DOUBLE–DIFFUSIVE CONVECTION)

Various observations in nature and especially in the ocean have developed in the field of convection giving rise to a range of new phenomenon by the consideration of the simultaneous presence of two or more components with different molecular diffusivities instead at the special variation of a single diffusing property such as heat in the classical thermal instability problems. Buoyancy forces can arise not only from density differences but also due to variations in solute concentration. Such problems arise in oceanography and engineering. Examples of particular interest are provided by some Antarctic lakes and ponds build to trap solar heat.

Intuition based on simple thermal convection can be misleading here. For example, in many system of interest, instabilities can develop even when the net density decreases upwards, and thus the system would be judged hydrostatically stable in a single component fluid. Diffusion, which has a stabilizing influence in a fluid containing a single solute, can in the double or multiple component case act to release the potential energy in the component that is heaviest at the top and convert it into the kinetic energy of the motion . Since the ideas concerned with the ocean in mind where heat and salt (or some other dissolved substances) can be dynamically important, therefore the process has been called thermohaline (or thermosolutal) convection. This phenomenon is also known as double-diffusive convection.

The form of the instability in a double diffusion system depends on the relation between the diffusivities and the density gradients i.e. on whether the driving energy is provided by the substance of higher or lower diffusivity. If the faster diffusing component is unstably stratified, convective motions occur in a manner more similar to thermal convection, this case is called 'Diffusive'. On the other hand, if it is the slower diffusing component, the convection typically takes the form of long vertical cells and this class is called 'finger' convection.

1.4. FERROHYDRODYNAMICS

Ferromagnetic fluid is a kind of liquid, which can be characterized as a colloidal system of sufficiently small mono-domain magnetic particles (of mean diameter of about 10 nm) dispersed in different carrier liquids. These fluids behave as a homogeneous continuum and exhibit a variety of interesting phenomenon. These particles are coated with a stabilizing dispersing agent (surfactant) which prevents particle agglomeration even when a strong magnetic field gradient is applied to the ferromagnetic fluid. The resulting material behaves like a normal fluid except that it can experience forces due to magnetic polarization. Ferromagnetic fluids are not found in nature but are artificially synthesized. The importance of ferrofluids was realized soon after the method of formation of ferrofluids, during mid sixties of the twentieth century. This is because it had a very large potential application in various fields, however due to the availability of colloidal magnetic fluids, many other uses of these fascinating liquids have been identified which are governed with a remote positioning of magnetic field controlling the magnetic fluid.

Due to the availability of colloidal magnetic fluids (ferrofluids) many uses of these fascinating liquids have been identified, which are concerned with the remote positioning and control of the magnetic fluid using magnetic force fields. Demonstrated applications of ferrofluids span a very wide range e.g. in instrumentation, lubrication, printing, vacuum technology, vibration dumping, metal recovery and medical science; and its commercial usage includes novel zero-leakage rotatory shaft seals in computer drives Bailey (1983).

In essence, thus, we can say that magnetic fluids are a unique class of materials. Ferrofluid technology is well established and capable of solving a wide variety of technical problems. There are many successful applications of this engineering material and there is an immense scope of further research.

The basic mathematical equations governing a flow in a ferrofluid layer which is under the simultaneous action of a uniform vertical magnetic field $\vec{H}$ and a uniform rotation $\vec{\Omega}$ about the vertical are as follows Vaidyanathan (2002):

The equation of continuity is:

$$\nabla.\vec{q} = 0 \tag{1}$$

The equation of motion is:

$$\rho_0\left(\frac{\partial\vec{q}}{\partial t} + \vec{q}\,.\nabla\,\vec{q}\right) = -\nabla p + \rho\vec{g} + \nabla.\left(\vec{H}\,\vec{B}\right) + \eta\nabla^2\vec{q} + 2\rho_0\left(\vec{q}\times\vec{\Omega}\right) + \frac{\rho_0}{2}\nabla\left(\left|\vec{\Omega}\times\vec{r}\right|^2\right) \tag{2}$$

The temperature equation is:

$$\left[\rho_0 C_{V,H} - \mu_0\vec{H}.\left(\frac{\partial\vec{M}}{\partial T}\right)_{V,\vec{H}}\right]\frac{dT}{dt} + \mu_0 T\left(\frac{\partial M}{\partial T}\right)_{V,\vec{H}}.\frac{d\vec{H}}{dt} = K_1\nabla^2 T + \phi, \tag{3}$$

where the dynamical quantities used are $\vec{q} = (u, v, w)$ velocity of ferrofluid, $p = p' - \rho_0\nabla\left(\left|\vec{\Omega}\times\vec{r}\right|^2\right)$ pressure, $\vec{g}$ = (0, 0, − g) acceleration due to gravity, $\vec{H} = (0,0,H_0)$ magnetic field, M magnetization, K_1 thermal conductivity, ϕ viscous dissipation factor, $C_{V,H}$ specific heat at constant volume and constant magnetic field, $\rho = \rho_0[1 + \alpha(T_0 - T)]$ density of the fluid where ρ_0 is density of the fluid at the reference temperature T_0 and μ_0 magnetic permeability of free space.

The linearized magnetic equation of state is :

$$M = M_0 + \chi(H - H_0) - k_2(T - \overline{T}_a) \tag{4}$$

where M_0 is the magnetization when magnetic field is H_0 and temperature $\overline{T}_a$, $\chi = \left(\frac{\partial M}{\partial H}\right)_{H_0,\overline{T}_a}$ is magnetic susceptibility, $k_2 = -\left(\frac{\partial M}{\partial T}\right)_{H_0,\overline{T}_a}$ is pyromagnetic coefficient, and $\overline{T}_a$ is the average temperature.

1.5. POROUS MEDIUM

The convective motion in porous media has been subject of intensive study because of its importance in the prediction of ground water movement in aquifers, in the energy extraction process from the geothermal reservoirs, in assessing the effectiveness of fibrous insulations and in nuclear engineering.

A medium which is solid body containing pores is called porous medium. 'External small void spaces in a solid are called molecular interstices' and 'very large ones are called caverns'. Pores are void spaces intermediate in size between caverns and interstices. Flow of fluid is possible only if at least a part of pore space is interconnected. The interconnected part of pore system is called effective pore space of the porous medium. The porosity ϕ of a porous medium is defined as the fraction of the total volume of the medium that is occupied by void space. Thus $1-\phi$ is the fraction that is occupied by solid.

Thus, $$\phi = \frac{\text{Volume of pores}}{\text{Bulk volume}} \quad . \tag{5}$$

Permeability of the porous medium characterizes the ease with which a fluid may be made to flow through the medium by an applied pressure gradient. If horizontal linear flow of an incompressible fluid is established through a sample of porous material of length L in the direction of flow, and cross-sectional area A, then the permeability K, of the material is defined as

$$K = \frac{q\mu}{A(\frac{\Delta P}{L})} \quad . \tag{6}$$

Here q is the fluid flow rate in volume per unit time, μ is the viscosity of the fluid and ΔP is the applied pressure difference across the length of the specimen. The porous medium of moderately large permeability necessitates the use of the Brinkman's (1947a, b) model and medium of very low permeability allows us to use the Darcy's (1856) model.

The basic hydrodynamic equations that govern the flow in porous medium are given by (Wankat and Schowalter (1970)):

The continuity equation for an incompressible fluid is

$$\frac{\partial u_j}{\partial x_j} = 0 \tag{7}$$

Equation of motion is

$$\rho\left(\frac{\partial u_i}{\partial t} + u_j\frac{\partial u_i}{\partial x_j}\right) = -\frac{\partial p}{\partial x_i} - \frac{\mu\epsilon}{k_1}u_i + \rho X_i \tag{8}$$

Equation of heat conduction is

$$A\frac{\partial T}{\partial t} + u_j\frac{\partial T}{\partial x_j} = \kappa\nabla^2 T \tag{9}$$

Equation of mass diffusion is

$$\frac{\partial S}{\partial t} + u_j\frac{\partial S}{\partial x_j} = \kappa_S\nabla^2 S \tag{10}$$

Equation of state is

$$\rho = \rho_0[\,1 + \alpha(T_0 - T) - \alpha'(S_0 - S)] \tag{11}$$

In the above equations ρ is the density, t is time, x_j (j=1,2,3) are the cartesian co-ordinates, u_j (j=1,2,3) are the velocity components, X_i (i=1,2,3) are the components of external force, p is the pressure, μ is the fluid viscosity, k_1 is the permeability of the porous medium, $\kappa = \frac{k'}{\rho_0 \epsilon c_0}$ is thermal diffusivity, where k' is thermal conductivity of the fluid-solid mixture, c_o is heat capacity of the fluid, ϵ is the porosity of the medium, κ_S is the effective mass diffusivity in the porous medium, T is temperature, S is the concentration, α and α' are respectively the coefficients of volume expansion due to temperature variation and concentration variation. Eq. (9) involves one more constant $A = 1 + \frac{\rho_{s_0} c_{s_0}}{\rho_0 c_0}\frac{1-\epsilon}{\epsilon}$ where ρ_{s_0} is the solid density, c_{s_0} is the heat capacity of the solid. The suffix '0' denotes the values of various parameters involved in the governing equations at some properly chosen mean temperature T_o.

The increasing demand for oil, water and food produced in an environmentally sound manner have placed emphasis on the manner of their production, a major part of which is concerned with flow through porous medium. Flow through porous medium is also of interest in chemical engineering (adsorption, filtration, flow in packed columns), in the petroleum engineering, in hydrology, in soil physics, in bio physics and in geophysics (Neild and Bejan (2006), Straughan (2008)).

2. REVIEW OF LITERATURE

2.1. THERMAL CONVECTION

The onset of thermal convection in a fluid layer heated from below, generally known as Rayleigh-Benard convection, has been extensively studied by many researchers and is well suited to illustrate many facets, mathematical and physical, of the general theory of hydrodynamical stability. Benard (1900) through his experiments demonstrated the onset of instability in fluids. He showed that a thin layer of fluid becomes unstable when the temperature difference exceeds a minimum (which depends strongly on the depth and less strongly on the properties of the fluid) and that the motions ensure on surpassing the minimum have a stationary cellular character. Later on, Lord Rayleigh (1916) laid a theoretical and mathematical foundation for the correct interpretation of the results of Benard's experiments. He showed that what decides the stability, or otherwise, of a layer of fluid heated from below is the numerical value of the non-dimensional parameter,

$$R = \frac{g\alpha\beta d^4}{\kappa\nu},$$

instability must set in when R exceeds a certain critical value R_c ; and that when R just exceeds R_c , a stationary pattern of motions must come to prevail.

Boussinesq (1903) suggested that there are many situations of practical occurrence in which basic equations can be simplified considerably. The situations occur when the variability in the density and in other coefficients due to variations in temperature are of moderate amounts.

Further remarkable contributions to the problem of Rayleigh-Benard convection (thermal instability) are due to Jeffreys (1926, 1928), Low (1929) and Pellew and Southwell (1940), Schmidt and Milverton (1935), Schmidt and Saunders (1938), Chandrasekhar (1952), Banerjee and Gupta (1991), Drazin and Reid (1981), Gupta and Dhiman (2001), Laroze and Martinez-Mardones (2007), Pardo, Herrero and Hoyas (2011) and Getling (2012). For a broader view of the subject on Rayleigh-Benard convection one may be referred to Chandrashekhar (1961).

2.2. THERMOHALINE CONVECTION

Stern (1960), has treated the stability of horizontal layer of fluid which is heated from above and in which the mass concentration of a chemical dissolved is maintained at S_0 at lower boundary and S_1 at the upper boundary ($S_1 > S_0$). The temperature, at the two boundaries are maintained at T_0 and T_1 respectively, with $T_0 < T_1$ whereas Veronis (1965) has studied a configuration in which $T_0 > T_1$ and $S_0 > S_1$. In other words he investigated the stability of a horizontally infinite layer of fluid which is subjected to a stabilizing salt gradient and to a destabilizing temperature gradient. The analysis of Veronis indicates that the system should become unstable to overstable perturbations when the disturbance is infinitesimal.

The subsequent work on the problem has been done by Sani (1965), Nield (1967), Shirtcliff (1969), Hurl and Jakeman (1971), Turner (1968, 1973, 1974), Huppert (1976), Huppert and Turner (1981), Bhattacharjee (1987), Yih (1980), Yao and Rogers (1989), Brandt and Fernando (1996), Gupta et al. (2001, 2002), Awad et. al. (2010). Some of these investigations bear ample theoretical evidence that overstability is the preferred mode of setting in of instability in the system. When instability sets in as overstability and one or both the boundaries are rigid, exact solution of the governing equations in closed form are not obtainable. Therefore it becomes important to drive bounds for the complex growth rate of an arbitrary oscillatory perturbation neutral or unstable. Banerjee et.al (1981) proposed a new scheme of combining the governing differential equation of the problem and showed that the complex growth rate 'n' of an arbitrary oscillatory perturbation neutral or unstable lies inside a semi circle in the right half of the complex n-plane. This result is shown to hold for all combination of rigid or free boundaries. The above result of Banerjee et.al has further been generalized and extended by Gupta.et.al (1982, 83, 84). An excellent review of the work on thermohaline convection with special reference to the fields of applications has been written by Turner (1973, 74), Huppert and Turner (1981) and Chen and Johnson (1984).

The convective motion in porous media has been subject of intensive study because of its importance in the prediction of

- ground water movement in aquifers,
- in the energy extraction process from the geothermal reservoirs,
- in assessing the effectiveness of fibrous insulations and
- in nuclear engineering.

The stability of flow of a fluid through porous medium was studied by Lapwood (1948) and Wooding (1960). Tountan and Lightfoot (1972) characterized salt fingers in thermohaline convection in porous medium. Thermal modulation of Rayleigh-Benard convection in a sparsely packed porous medium is studied by Bhadauria (2007). The problem of double diffusive convection in porous medium has been extensively investigated and the growing volume of work to this area is well documented by Ingham and Pop (2005), Nield and Bejan (2006) and Vafai (2006) and Straughan (2008).

2.3. FERROCONVECTION

The convective instability of a ferromagnetic fluid for a fluid layer heated from below in the presence of uniform vertical magnetic field has been considered by Finlayson (1970). Thermoconvective stability of ferrofluids without considering buoyancy effects has been investigated by Lalas and Carmi (1971), whereas Shliomis (1974) analyzed the linearized relation for magnetized perturbed quantities at the limit of instability. Schwab et al. (1983) investigated experimentally the Finlayson problem in the case of a strong magnetic field and detected the onset of convection by plotting the Nusselt number versus the Rayleigh number. Then, the critical Rayleigh number corresponds to a discontinuity in the slope. Later, Stiles and Kagan (1990) examined the experimental problem reported by Schwab et al. (1983) and generalized the Finlayson's model assuming that under a strong magnetic field the rotational viscosity augments the shear viscosity. Jyoti Prakash (2012) derived a sufficient condition for the validity of the exchange principle and upper bounds for the complex growth rate of an arbitrary oscillatory perturbation which may be neutral or unstable in a ferrofluid layer heated from below. For the broad view of the subject one is referred to Rosenweig (1985) and Odenbach (2002).

3. RESEARCH METHODOLOGY

Mathematically the stability analysis of a problem in hydrodynamics proceeds along the following lines:

(i) First of all initial basic partial differential equations describing the system are written.

(ii) Initial state solutions (no motions are there) are obtained.

(iii) The system is then slightly perturbed, and the physical quantities describing the flow suffer small increments, the perturbation equations governing these increments are then obtained,

(iv) The perturbation equations obtained in step (ii) above are then linearized by ignoring the second and higher order terms in the equations on the lines of the linear stability theory, then

(v) The normal mode analysis method is used to determine the stability of a stationary state of a hydrodynamic or hydromagnetic system in the present work. The beauty of this method is that it gives complete information about instability including the rate of growth of any unstable perturbation. This method has been used throughout by Chandrasekhar in his book "Hydrodynamic and Hydromagnetic Stability" (Dover Publication, New York, [1961]), while discussing the various instability problems.

To solve these equations we assume that the perturbed quantities have time variations proportional to e^{nt}. The parameter 'n' is, in general, function of k (the wave-number) and of other parameters defining the system. If the value of n determined by the dispersion relation is:

(i) Real and negative, the system is stable;

(ii) Real and positive, the system is unstable;

(iii) Complex, say, $n = n_r + in_i$, where n_r and n_i are real.

Then the following cases can arise:

(I). if $n_r < 0$, the system is stable,

(II). if $n_r > 0$, the system is unstable,

(III). if $n_r = 0$, we have a state of neutral stability,

(IV). further, if $n_r = 0$ implies that $n_i = 0$, then the stationary (cellular) pattern of flow prevails on the onset of instability. In other words, the "principle of exchange of stabilities" is valid,

(V). if $n_r = 0$ does not imply that $n_i = 0$, then overstability occurs.

From this, it follows that if n is real, then n = 0 will separate the stable and unstable modes and there will always be exchange of stabilities.

After the normal mode analysis is applied on the linear perturbation equations, the partial differential equations are converted to the ordinary differential equations, which are easier to solve. These equations are then non-dimensionalized using non-dimensional numbers:

Rayleigh number $R = \frac{g\alpha\beta d^4}{\kappa_T \nu}$, Thermohaline Rayleigh number $R_S = \frac{g\gamma\delta d^4}{\kappa_T \nu}$, Chandersekhar number $Q = \frac{\mu H^2 d^2}{4\pi\rho_0 \nu \eta}$, Prandtl number $P_r = \frac{\nu}{\kappa_T}$, Darcy number $D_a = \frac{k_1}{d^2}$, $D = d\frac{d}{dz}$, a = k d etc.

Then using bilinear forms method, analytical results regarding obtaining the bounds on the Eigen values of the problem in terms of the flow parameters have been obtained. Further, sufficient conditions for the holding of stationary convection have been obtained. From the analytical analysis some characterization theorems regarding the non existence of oscillatory motions in initially bottom heavy configurations of some thermohaline convection configurations in porous medium have been obtained.

4. THEORETICAL /EXPERIMENTAL BACKGROUND

H. Benard through his experiments, first of all, demonstrated the onset of thermal convection in a fluid layer heated from below. Theoretical foundations for the above mentioned phenomena were laid by Lord Rayleigh (1916). Further remarkable contributions to the problem of thermal convection are due to Jeffreys (1926, 1928), Low (1929) and Pellew and Southwell (1940), Schmidt and Milverton (1935), Schmidt and Saunders (1938), Chandrasekhar (1952), Banerjee and Gupta (1991), Drazin and Reid (1981), Pardo, Herrero and Hoyas (2011) and Getling (2012).

In the 1960s, scientists from the National Aeronautics and Space Administration (NASA) research centre investigated methods for controlling liquids in space. They developed ferrofluids, which are colloidal suspension of magnetic nano-particles in a carrier fluid such as water, hydrocarbons or fluorocarbons. There have been numerous studies on thermal convection in a

ferrofluid layer called ferroconvection analogous to Rayleigh-Benard convection in ordinary viscous fluids. Some theoretical and experimental contributions to the problem of ferroconvection are due to Rosenweig (1985), Lalas and Carmi (1971), Schwab et. al. (1983) and Borglin et al. (2000).

5. MOTIVATION/OBJECTIVES OF THE PROPOSED RESEARCH WORK

Two fundamental configurations have been studied in thermohaline instability, the first one by Stern (1960) wherein the temperature gradient is stabilizing while the concentration gradient is destabilizing and the second one by Veronis (1965) wherein the temperature gradient is destabilizing while the concentration gradient is stabilizing.

Banerjee et al. (1993) characterized, in the parameter space of the system alone, the nonexistence of oscillatory motions of growing amplitude in an initially bottom heavy Veronis thermohaline configuration and the following result is obtained in this direction namely that thermohaline convection of the Veronis type cannot manifest itself as oscillatory motions of growing amplitude in an initially bottom heavy configuration if the thermohaline Rayleigh number R_S, the Lewis number τ, and the Prandtl number σ satisfies the inequality

$$R_S \leq \frac{27}{4}\pi^4\left(1+\frac{\tau}{\sigma}\right).$$

A similar theorem is also derived for thermohaline convection of the Stern type. The above characterization theorems of Banerjee et al. have brought a fresh outlook to the subject matter of double diffusive convection and paved the way for further theoretical and experimental investigations in this field of enquiry. The essence of Banerjee et al.'s characterization theorems lie in that it provides a classification of the neutral and unstable thermohaline convection of the Veronis and Stern type into two classes, the bottom heavy class and the top heavy class and then strikes a distinction between them by means of characterization theorems which disallow the existence of oscillatory motions in the former class.

Flow through porous medium is a topic encountered in many branches of engineering and science like, hydrology, reservoir engineering, soil science, soil mechanics and chemical engineering (Neild and Bejan (2006), Straughan (2008)).

Jyoti Prakash and Vinod Kumar (2010, 2011) have extended the Banerjee et al.'s (1993) work to Porous medium and derived characterization theorems for the nonexistence of oscillatory motions of growing amplitude in an initially bottom heavy configuration of Veronis type and Stern type. The extension of these two important theorems to the domain of astrophysics and terrestrial physics, wherein the liquid concerned has the property of electrical conduction and magnetic fields and rotations are prevalent, is very much to be sought for in the present context and in this thesis we undertook a detailed investigation of the effects of a uniform rotation / uniform magnetic field about an axis parallel to gravity on these characterization theorems in hydrodynamics in porous medium.

The problem of obtaining a sufficient condition for the validity of the principle of exchange of stabilities and bounds for the complex growth rate of an arbitrary oscillatory motion of growing amplitude in a ferroconvection configuration is important especially when both the boundaries are not dynamically free so that solutions in the closed form are not obtainable. Jyoti Prakash (2012) derived a sufficient condition for the validity of the exchange principle and upper bounds for the complex growth rate of an arbitrary oscillatory perturbation which may be neutral or unstable in a ferrofluid layer heated from below. As a further step these problems have been extended in the present work with magnetic field dependent viscosity.

6. PLAN OF WORK WITH WORK DONE TILL DATE

The aim of the present work is to analyze the stability of thermohaline convection in a porous medium in the following cases:

(i) when no magnetic field or rotation is applied, or
(ii) when a uniform rotation about the vertical is applied, or
(iii) when a uniform vertical magnetic field is applied, or
(iv) when both a uniform rotation about the vertical and a uniform vertical magnetic field are simultaneously applied.

We also propose to analyze the stability of a ferrofluid layer heated from below in a porous or non-porous medium which is

(i) under the action of uniform vertical magnetic field, or

(ii) under the simultaneous action of a uniform vertical magnetic field and a uniform rotation about the vertical.

Linear stability behavior of the above mentioned problems has typically been examined through mathematical analysis by considering all the cases of different boundaries, whether rigid or dynamically free and general qualitative results have been obtained.

The proposed thesis shall be divided tentatively into five chapters and will represent our attempt to mathematical investigations of thermohaline convection and ferromagnetic convection in the framework of linear stability theory.

Chapter 1 is introductory in nature and concises the essential results of stability analysis in the respective fields of ferromagnetic convection, rotatory ferromagnetic convection, thermohaline convection, rotatory thermohaline convection, magnetic thermohaline convection and magnetorotatory thermohaline convection in porous or non-porous medium relevant to the thesis. This introductory chapter effectively lays down the fundamental framework on which the more general investigation presented in the subsequent chapters rest upon.

Chapter 2 is primarily motivated by the work of Banerjee et al. (1993). In this chapter a thermohaline convection configuration of the Veronis and Stern type (neutral or unstable) in porous medium have been classified into two classes namely, the bottom heavy class and the top heavy class and then strikes a distinction between them by means of characterization theorems which disallows the existence of oscillatory motions in the former class. The effects of magnetic field, rotation or magnetic field and rotation simultaneously on the above configurations are also studied. The following results are obtained in this direction:

(i) rotatory thermohaline convection of the Veronis type in porous medium cannot manifest itself as oscillatory motion of growing amplitude in an initially bottom heavy configuration if the thermohaline Rayleigh number R_S, the Lewis number τ, the Prandtl number p_1, the porosity ϵ, satisfy the inequality $R_S \leq 4\pi^2 \left(\frac{1}{P_l} + \frac{\tau}{E'\epsilon\, p_1}\right)$, where P_l and E' are constants which depend upon porosity of the medium. It further establishes that this result is uniformly valid for the quite general nature of the

bounding surfaces. A similar characterization theorem is also proved for rotatory thermohaline convection of the Stern type.

(ii) magnetothermohaline convection of the Veronis type in porous medium cannot manifest as oscillatory motion of growing amplitude in an initially bottom heavy configuration if the thermohaline Rayleigh number R_S , the Lewis number τ, the Prandtl number p_1, the porosity ϵ, satisfy the inequality $R_S \leq \frac{4\pi^2}{P_l} + \frac{27\,\pi^4}{4}\,\frac{\tau}{E'\epsilon\, p_1}$, where P_l and E' are constants which depend upon porosity of the medium. It further establishes that this result is uniformly valid for any combination of rigid or free perfectly conducting bounding surfaces. A similar characterization theorem is also proved for magnetothermohaline convection of the Stern type.

(iii) magnetorotatory thermohaline convection of the Veronis type in porous medium cannot manifest itself as oscillatory motions of growing amplitude in an initially bottom heavy configuration if the thermohaline Rayleigh number R_S, the Lewis number τ, the Prandtl number P_r, the porosity ϵ, satisfy the inequality $R_S \leq 4\pi^2\left(\frac{1}{D_a} + \frac{\tau}{E'P_r\epsilon}\right)$, where D_a the Darcy number and E' are constants which depend upon porosity of the medium. It further establishes that this result is uniformly valid for any combination of rigid or free perfectly conducting bounding surfaces. A similar characterization theorem is also proved for magnetorotatory thermohaline convection of the Stern type.

(iv) thermohaline convection of the Veronis type in porous medium (Darcy-Brinkman Model) cannot manifest itself as oscillatory motions of growing amplitude in an initially bottom heavy configuration if the thermohaline Rayleigh number R_S, the Lewis number τ, the Prandtl number p_1, the porosity ϵ, satisfy the inequality $R_S \leq \frac{27}{4}\pi^4\left(\Lambda + \frac{\tau}{E'p_1\epsilon}\right) + \frac{4\pi^2}{P_l}$, where P_l and E' are constants which depend upon porosity of the medium and $\Lambda = \frac{\mu_e}{\mu}$, here μ_e is the effective viscosity and μ is the dynamic viscosity of the fluid. It further establishes that this result is uniformly valid for the quite general nature of the bounding surfaces. A similar characterization

theorem is also proved for magnetorotatory thermohaline convection of the Stern type.

(v) rotatory thermohaline convection of the Veronis type in porous medium (Darcy-Brinkman Model) cannot manifest itself as oscillatory motions of growing amplitude in an initially bottom heavy configuration if the thermohaline Rayleigh number R_S, the Lewis number τ, the Prandtl number P_r, the porosity ϵ, satisfy the inequality $R_S \leq \frac{27\pi^4}{4}\left(\Lambda + \frac{\tau}{E' P_r \epsilon}\right) + 4\pi^2 D_a^{-1}$, where D_a is the Darcy number, E' is a constant which depends upon the porosity of the medium and $\Lambda = \frac{\mu_e}{\mu}$, here μ_e is the effective viscosity and μ is the dynamic viscosity of the fluid. An analogous theorem for rotatory thermohaline convection of the Stern type in porous medium is also proved.

(vi) magnetothermohaline convection of the Veronis type in porous medium (Darcy-Brinkman Model) cannot manifest itself as oscillatory motion of growing amplitude in an initially bottom heavy configuration if the thermohaline Rayleigh number R_S, the Lewis number τ, the Prandtl number P_r, the porosity ϵ, satisfy the inequality $R_S \leq \frac{27\pi^4}{4}\left(\Lambda + \frac{\tau}{E' P_r \epsilon}\right) + 4\pi^2 D_a^{-1}$, where D_a is the Darcy number and E' is a constant which depend upon porosity of the medium and $\Lambda = \frac{\mu_e}{\mu}$, here μ_e is the effective viscosity and μ is the dynamic viscosity of the fluid. It further establishes that this result is uniformly valid for any combination of rigid or free perfectly conducting bounding surfaces. A similar characterization theorem is also proved for magnetothermohaline convection of the Stern type.

Chapter 3 is primarily motivated by the work of Banerjee et al. (1993) and Gupta et al. (2001). In this chapter we have obtained upper bounds for the oscillatory motions of neutral or growing amplitude in the thermohaline configurations of the Veronis and Stern types in porous medium in such a way that it also results in sufficient conditions of stability for an initially top heavy as well as initially bottom heavy configuration.

Chapter 4 is primarily motivated by the work of Finlayson (1970) and Jyoti Prakash (2012). In this chapter the problem of onset of convective instability in a horizontal layer of ferrofluid heated from below is examined under the effect of uniform rotation about the vertical and a sufficient condition for the validity of the principle of exchange of stabilities is derived. In the same chapter upper bounds for the complex growth rates in ferromagnetic convection with magnetic field dependent viscosity in a rotating ferrofluid layer heated from below are also obtained. The following results are obtained in this direction:

> the complex growth rate $\omega = \omega_r + \omega_i$ (ω_r and ω_i are respectively the real and imaginary parts of ω) of an arbitrary oscillatory motion of growing amplitude in ferromagnetic convection, with magnetic field dependent viscosity, in a rotating ferrofluid layer for the case of free boundaries, must lie inside a semicircle in the right half of the $\omega_r\omega_i$ - plane whose centre is at the origin and $(radius)^2 = max.\left\{\frac{RM_1}{P_r}, T_a\right\}$, where R is the Rayleigh number, M_1 is the magnetic number, P_r is the Prandtl number and T_a is the Taylor number. Further, bounds for the case of rigid boundaries are also derived separately.

Chapter 5 In this chapter the problem of onset of convective instability in a ferrofluid layer in a porous medium heated from below with magnetic field dependent viscosity is examined and upper bounds for the complex growth rate of an arbitrary perturbation, which may be neutral or unstable, are obtained. The following results are obtained in this direction:

> the complex growth rate $\omega = \omega_r + i\,\omega_i$ (ω_r and ω_i are respectively the real and imaginary parts of ω) of an arbitrary oscillatory motion of growing amplitude in ferromagnetic convection, with magnetic field dependent viscosity, in a porous medium for the case of free boundaries, must lie inside a semicircle in the right half of the $\omega_r\omega_i$ - plane whose centre is at the origin and $radius = \left(\frac{RM_1}{P_r}\right)^{\frac{1}{2}}$, where R is the Rayleigh number, M_1 is the magnetic number and P_r is the Prandtl number. Further, bounds for the case of rigid boundaries are also derived separately.

7. WORK TO BE DONE

In the light of Chapter 2 we also wish to mathematically analyze magnetothermohaline convection and magnetorotatory thermohaline convection of the Veronis and Stern types in the porous media by using Darcy-Brinkman model.

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RESEARCH PAPERS COMMUNICATED FOR PUBLICATION

1. Thermohaline convection of Veronis and Stern types in porous medium: Revisited.
2. Upper Bounds to the Complex Growth Rates in Ferromagnetic Convection with Magnetic Field Dependent Viscosity in a Porous Medium.

3. On arresting the complex growth rates in ferromagnetic convection with magnetic field dependent viscosity in a rotating ferrofluid layer.
4. On Exchange of Stabilities in Ferromagnetic Convection in a Rotating Ferrofluid Layer.
5. On Non-Existence of Oscillatory Motions in Rotating Thermohaline Convection in Porous Medium: Darcy-Brinkman Model.
6. On Non-Existence of Oscillatory Motions in Magnetothermohaline Convection in Porous Medium.

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